Derivation of Covered Interest Parity

 For the last two chapters of the textbook we have been investigating the causes of variation in international exchange rates. There are two basic determinants of this variation. One is due to international flows of goods and services. If there is a price difference for similar goods between countries, the cheapest good is the one that will be purchased and the currency of the country producing the cheapest good will be purchased. So, one determinant of exchange rates has to do with price differentials.

However, another possibility exists. If interest rates in one country exceed those in another, it might be possible to profit by borrowing funds in the low interest rate country and simultaneously lending them in the high interest rate country. This will work if the exchange rate doesn’t move to offset the interest differential. If this possibility exists, the currency of the high interest rate country will be purchased with the currency of the low interest rate country thereby affecting the exchange rate between the two countries. The purpose of this derivation is to find the condition whereby this kind of exchange would not be possible, commonly called covered interest parity.

For this derivation we will use the U.S. and the U.K. and a 90 day or 3-month investment of X dollars. The variables and definitions used in this derivation are:

 iU.S. = interest rate in the U.S.

 iU.K. = interest rate in the U.K.

 f = 90 day forward rate of the £ ($/£) (*i.e.* the 90 day forward dollar price of a pound)

 e = spot price of the £ or $/£ (*i.e.* the dollar price of a pound)

 1/e = spot price of the $ or £/$ (*i.e.* the pound price of a dollar).

Consider a U.S. investor who has X dollars to invest in either New York or London. The investment would be undertaken in New York if:

 $X(1 + i\_{U.S.}) > X\left[\frac{1}{e}\right](1 + i\_{U.K.})f$

The investor would invest in London when:

 $X\left(1 + i\_{U.S.}\right)< X\left[\frac{1}{e}\right](1 + i\_{U.K.})f$

The left-hand side of the equation—$X\left(1 + i\_{U.S.}\right)$—represents the future value of $1 in 90 days invested at the current U.S. interest rate. The term on the right-hand side that might be somewhat puzzling is $X\left[\frac{1}{e}\right]$. Recall that e = $/£ (the dollar price of a pound). So, to get the number of pounds that X dollars will buy, X is divided by e (the dollar price of a pound). [Keep in mind that to calculate the quantity of anything that a certain amount of money will buy you must divide by the price. In this case we need to calculate the number of pounds that X dollars will buy. So, we divide X by the price of pounds. Currently, the spot price of the pound is about $2.00. So, X dollars would buy about X/2 pounds.] In other words, X dollars is used to buy as many pounds as the market allows and that amount of pounds is invested at the market interest rate in London. Simultaneously, the principal and interest in pounds that will be available at the end of 90 days (the proceeds from the 90 day investment) is immediately sold at the current forward rate for pounds. [This is the cover that guarantees variation in the exchange rate during the 90 day investment period will not offset the return on the investment.] At the end of 90 days, the proceeds in pounds are converted back to dollars at the contracted 90 day forward rate (f).

Covered interest parity occurs when the two sides of the equation are equal:

$X\left(1 + i\_{U.S.}\right)= X\left[\frac{1}{e}\right](1 + i\_{U.K.})f$.

The above is an exact form of covered interest parity, but there is a very handy approximation which is easier to calculate and helps with interpretation. This approximation makes use of the forward premium. The forward premium (the forward exchange rate premium) is defined as:

 $F=\frac{(f-e)}{e}$ or $F=\left(\frac{f}{e}\right)-1$ . [If this calculation is made for a 90 day forward rate, it can be annualized simply by multiplying the rate by 4. In other words, if the calculated rate is 2%, it is 2% over 90 days which translates to 8% over a full year. This is helpful to know because most interest rates are stated on an annual basis.]

If F > 0, the dollar is at a forward premium and if F < 0 the dollar is at a forward discount. Using the forward premium as written above along with the covered interest parity condition a simpler result can be derived.

 $X\left(1 + i\_{U.S.}\right)= X\left[\frac{1}{e}\right](1 + i\_{U.K.})f$ [The covered interest parity equation]

First simplify by dividing both sides by X.

 $\left(1 + i\_{U.S.}\right)= \left[\frac{1}{e}\right](1 + i\_{U.K.})f$

Then combine the two exchange rate terms into a single term.

 $\left(1 + i\_{U.S.}\right)= \left[\frac{f}{e}\right]\left(1 + i\_{U.K.}\right)$

Now, subtract one from both sides.

 $i\_{U.S.}= \left[\frac{f}{e}\right]\left(1 + i\_{U.K.}\right)-1$

Because $\left(\frac{f}{e}\right)-1+1=\left(\frac{f}{e}\right)$, $\left(\frac{f}{e}\right)-1+1$can be substituted in the above equation for $\left(\frac{f}{e}\right)$giving

$i\_{U.S.}= \left[\left[\frac{f}{e}\right]-1+1\right]\left(1 + i\_{U.K.}\right)-1$.

Because $=\left(\frac{f}{e}\right)-1$ , the following equation can be derived by substituting F in the above equation:

$i\_{U.S.}= \left(F+1\right)\left(1 + i\_{U.K.}\right)-1$.

The above can be simplified by multiplying the two terms on the right hand side and subtracting the 1.

 $i\_{U.S.}= F+i\_{U.K.}+F\left( i\_{U.K.}\right)$

Because both F (the forward premium or discount) and $i\_{U.K.}$ are usually very small percentages and they are multiplied by each other, the resulting product is usually treated as roughly 0. So, the approximating equation for covered interest parity is:

 $i\_{U.S.}= F+i\_{U.K.}$ .

If the left-hand side exceeds the right-hand side, London investors move funds to the U.S. If the right-hand side exceeds the left-hand side, New York investors move funds to Britain.

The covered differential is derived by subtracting $i\_{U.S.}$ from both sides of the equation yielding:

 $CD= F+i\_{U.K.}-i\_{U.S.}$.

If CD > 0, there is an incentive for investment to move from the U.S to Britain. If CD < 0, there is an incentive for investment to move from Britain to the U.S. Of course, these movements have the predicted effect on the exchange rate and the forward premium.